



RF-3489
M. Sc. (Part-I) Examination
April / May – 2010
Paper - 403 : Topology

Time : 3 Hours]

[Total Marks : 70

Instructions :

(1)

<p>नीचे दृष्टावेक निशानीवाणी विगतो उत्तरवडी पर अवश्य लभवी. Fillup strictly the details of signs on your answer book.</p> <p>Name of the Examination :</p> <p>M. Sc. (Part-1)</p> <p>Name of the Subject :</p> <p>403 : Topology</p> <p>Subject Code No. : 3 4 8 9 Section No. (1, 2,.....): Nil</p>	<p>Seat No. :</p> <table border="1" style="width: 100%; height: 20px;"><tr><td style="width: 15%;"></td><td style="width: 15%;"></td><td style="width: 15%;"></td><td style="width: 15%;"></td><td style="width: 15%;"></td><td style="width: 15%;"></td></tr></table> <div style="border: 1px solid black; border-radius: 15px; height: 60px; margin-top: 10px; display: flex; align-items: center; justify-content: center; padding: 10px;">Student's Signature</div>						

(2) This paper contains **five** questions.

(3) All questions are **compulsory**.

(4) Each question is of (5+5+4) marks.

(5) Notations used are standard.

1 (a) State and prove Cauchy's inequality.

(b) Let X and Y be two metric space then prove that a mapping $f : X \rightarrow Y$ is continuous at $x_0 \in X$ iff

$$x_n \rightarrow x_0 \Rightarrow f(x_n) \rightarrow f(x_0)$$

(c) Let X be the metric space. If A and B are two sub-set of X then show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

OR

1 (a) Let X be a metric space and A, B are subset of X . then show the following :

(1) $Int(A) \cap Int(B) = Int(A \cap B)$

(2) $Int(A) \cup Int(B) \subset Int(A \cup B)$

- (b) Let $\{x_n\}$ and $\{y_n\}$ be sequences in a metric space X converging to x and y respectively then prove that $d(x_n, y_n)$ converges to $d(x, y)$,
- (c) Let X be a metric space. If A is subset of X then prove that $\overline{A^c} = (\text{Int}(A))^c$.

- 2** (a) Define Metrizable space. Let X be infinite set, then prove that co-finite topological space X is not Metrizable.
- (b) Let X is topological space. Let $A \subset X$ then prove that $\text{Bd}(A) = \emptyset \Leftrightarrow A$ is both open and closed.
- (c) Give an example of space which is not first countable.

OR

- 2** (a) State and prove Linderlof's theorem.
- (b) Define second countable space. Let X be second countable space. Then prove that any open base for X has a countable subclass, which is also an open base for X .
- (c) Let $Y \subset X$ where X is topological space and $Z \subset Y$, where Y is topological space then Z is subspace of X .

- 3** (a) State and prove Heine Borel theorem.
- (b) Prove that a closed subspace of a compact topo-space is compact.
- (c) Prove that any continuous map from compact metric space in to a metric space is uniformly continuous.

OR

- 3** (a) Define compact space. Prove that continuous image of a compact space is compact.
- (b) Define an open cover. Prove that every finite topological space is compact.
- (c) Prove that every sequentially compact $m \cdot s_p$ is compact.

- 4 (a) Define T_2 space. Prove that product of any non-empty class of T_2 space is a T_2 space.
- (b) Define T_1 space prove that every compact T_2 space is normal.
- (c) Prove that every T_2 space is T_1 space but converse is not true.

OR

- 4 (a) State and prove Uryshon's lemma.
- (b) Define Regular space. Prove that every compact H.d. space is regular.
- (c) Prove that every compact sub space of T_2 space is closed.
- 5 (a) Prove that a topo-space X is disconnected if and only if \exists continuous function f of X on to the discrete two pts space $\{0, 1\}$.
- (b) Prove that the continuous image of a connected space is connected.
- (c) Prove that product of two connected space X and Y is connected.

OR

- 5 (a) Prove that each point of topo-space X is contained in exactly one component of X .
- (b) Prove that a sub space of real line space R is connected if and only if it is interval.
- (c) With proper counter example, show that the component of a topo-space may not be open.